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# Mechanisms of dilatant en échelon crack formation in bedded chalk

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**Abstract**—This study focuses on various styles of dilatant out-of-plane en échelon cracks, which are associated with bed restricted burial joints, that cut slightly deformed sediments in the Beer Sheva syncline. There is a positive linear fit of en échelon crack length to layer thickness ($R^2 = 0.928$) in fringes which were formed by discontinuous breakdowns in joints. Lithological properties influence the breakdown process. The breakdown of joints into en échelon cracks in chalk beds is influenced by both local conditions and remote stresses. There are two distinct cases of opposite senses of segment rotation in a single fringe: first, in a curved fringe; and, second, in a straight fringe. The former can best be assigned to local deviations of the minimum principal direction from perpendicularity to the parent joint, whereas the latter appears to reflect changes in remote stresses. Two of the three sets of en échelon segmentation which were distinguished in the Beer Sheva syncline are correlated with two distinct tectonic associations. The $K_{III}/K_I$ ratio increases from the layer centre towards the layer boundaries, and this is the driving motivation for both the gradual increase of barb overlap from the centre towards the layer boundaries, and the en échelon segmentation along the layer boundaries. © 1997 Elsevier Science Ltd.

## INTRODUCTION

Interaction of non-coplanar cracks was observed by De Freminville (1914) in bitumin. Following that, Preston (1931) modelled the process leading to striae (and en échelon segmentation) in glass. He pioneered the explanation that when a change in the direction of the principal tension occurs the crack must breakdown into a number of segments so that each segment readjusts its orientation to maintain normality with respect to the new direction of the principal tension. This mechanism has since been reaffirmed (e.g. Sommer, 1969; Cooke and Pollard, 1996) and was formulated into a sophisticated fracture mechanics theory of mixed mode I + III operation (Lawn and Wilshaw, 1975; Pollard *et al.*, 1982; Cooke and Pollard, 1996).

En échelon segmentation is ubiquitous on various scales from microns to kilometres in geological exposures occurring in different terrains; implying that breakdowns may be the consequences of various mechanisms (see a comprehensive description of en échelon distribution in Pollard *et al.*, 1982).

In spite of the great importance and the rich literature on en échelon segmentation, many questions remain open regarding this process.

(1) There are many mixed mode I + III experimental studies on engineering materials (e.g. Gilman, 1958; Sommer, 1969; Hertzberg, 1976; Tschegg, 1983) and on *in situ* en échelon veins (e.g. Shainin, 1950; Hancock, 1972), but there are relatively few detailed field investigations on dilatant en échelon cracks in jointed layered rocks. There is a need for more field data for the elucidation of the relationships between parent joints and en échelon segments, the propagation of segments, their development in different styles and their dependence on the lithological properties of the rock.

(2) Especially intriguing is the problem of distinguishing between the influences of local stresses and remote stresses on the formation of en échelon cracks.

(3) The structural interpretation of en échelon cracks is still subject to debate (Engelder *et al.*, 1993, p. 235). There is a need for further clarification of the mixed mode mechanisms leading to segmentation, and their possible relationship to palaeostress directions (Younes and Engelder, 1995).

The present study is mostly related to fracture in Eocene chalk beds from the Beer Sheva syncline (Fig. 1) which are only slightly deformed platform cover sediments. The focus is on bed restricted burial joints, which are distinguished from syntectonic joints that form under greater energy conditions and often are not layer restricted (Bahat, 1991a). Chalk is a low strength rock with considerable changes in properties when exposed to aqueous solutions. Therefore, fracture is readily induced in this material, even at very low differential stress. Jointing in Eocene chalks around Beer Sheva is enriched with various phenomena of en échelon segmentation, which bear relationships to various fracture styles and multiple joint systems (Bahat, 1986, 1991a,b). Figure 2(a) is a histogram of the joint azimuths in an individual outcrop from the Lower Eocene in the Beer Sheva syncline. It shows the relatively low spread of joint orientation at the outcrop. On the other hand, Fig. 2(b) is a comparison of joint distribution from the Lower Eocene with that from the Middle Eocene in the Beer Sheva syncline (after Bahat and Grossman, 1988). It gives some idea of the complexity of the palaeostress directions (not only a situation of joint rotation) in the northern Negev, which is partly reflected in the occurrence of several distinct en échelon sets in the area. The objective of this

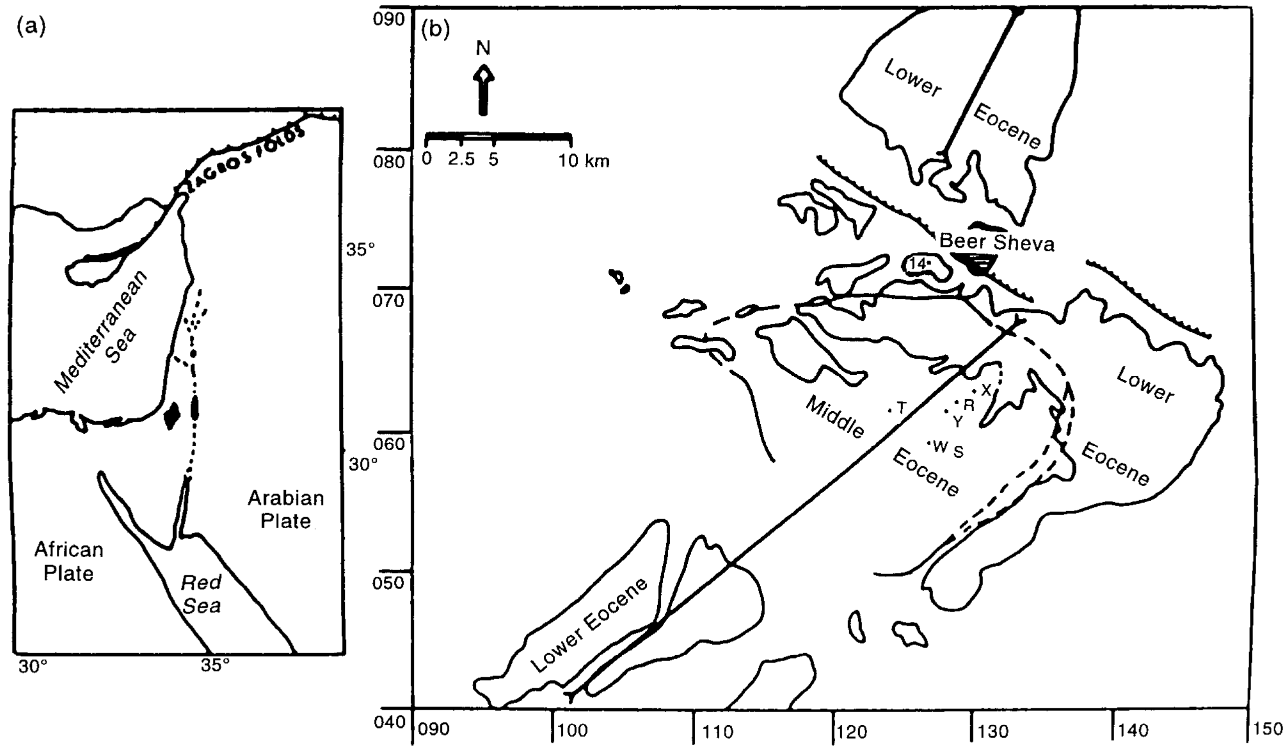



Fig. 1. (a) Location map showing the investigated area as a rhomb on the map. (b) Area map showing Beer Sheva at the centre, between the southern tip of the Shephela syncline in the north and the northern edge of the Beer Sheva syncline in the south. Straight heavy lines are portions of fold axes; curved heavy lines are boundaries of exposed Eocene rocks, after Bentor *et al.* (1970); and light lines (partly dashed) are boundaries between the Lower and Middle Eocene formations. Barbed lines are inferred faults south of Beer Sheva, after Gvirtzman (1969). Locations of outcrops are marked on the map (Emek Haela is beyond this map).

study is to report new observations, and to examine the three statements given above.

## FIELD OBSERVATIONS

Various styles of dilatant out-of-plane en écheon cracks in Eocene chalk are presented below. These are related to six outcrops in the Beer Sheva syncline (Fig. 1) and one outcrop from the Shefela syncline (north of Beer Sheva). With the exception of the first two outcrops, which are from the Lower Eocene, all the other exposures are in the upper beds of the Middle Eocene, which are the youngest chalks in the Beer Sheva syncline (the Israeli grid reference of each station is given in brackets). The en échelon segmentation styles in these exposures are different from the segmentation style of discoid joints in Cenomanian chalk from Mt Carmel in northern Israel.

*Emek Haela (1222/1450)*

A fracture-cutting Lower Eocene chalk in the Shefela syncline is shown in Fig. 3. En échelon fringes occur both on the upper and the lower sides of the parent joint. There is a clockwise rotation of the en échelon segments with respect to the parent joint in the upper fringe and a counterclockwise rotation in the lower one. The plume on the joint surface propagated from left to right, so that the segments in the lower fringe are in a rational continuation with the plume, but the segments in the upper fringe are not. Also, whereas the segments in the lower fringe are of approximately equal size, the segments in the upper fringe are of different sizes, generally larger than in the lower fringe. The segments in these two fringes also differ in shape.

*Station 14 (0723/1261)*

En échelon segmentation is almost absent in joints of the Lower Eocene around Beer Sheva. There is one exception (station 14; Bahat, 1991a, fig. 5.29) which occurs along sharp, welded contacts between massive beds of chert with chalk layers. This case represents an intense counterclockwise segmentation in the chalk along both the lower and upper boundaries of the chert bed (Fig. 4a & b). The segmentation occurs along both lower and upper boundaries of chalk layers. No breakdown to en échelon cracks occurs along contacts between chalk layers and beds of chert nodules in the area. One notes that the massive chert bed is jointed and the breakdown in the chalk is confined to the joint. Segmentation does not occur along lithological interfaces remote from joints in the chert. The sense of rotation of the segments in all the fringes is counterclockwise. The joints cutting the

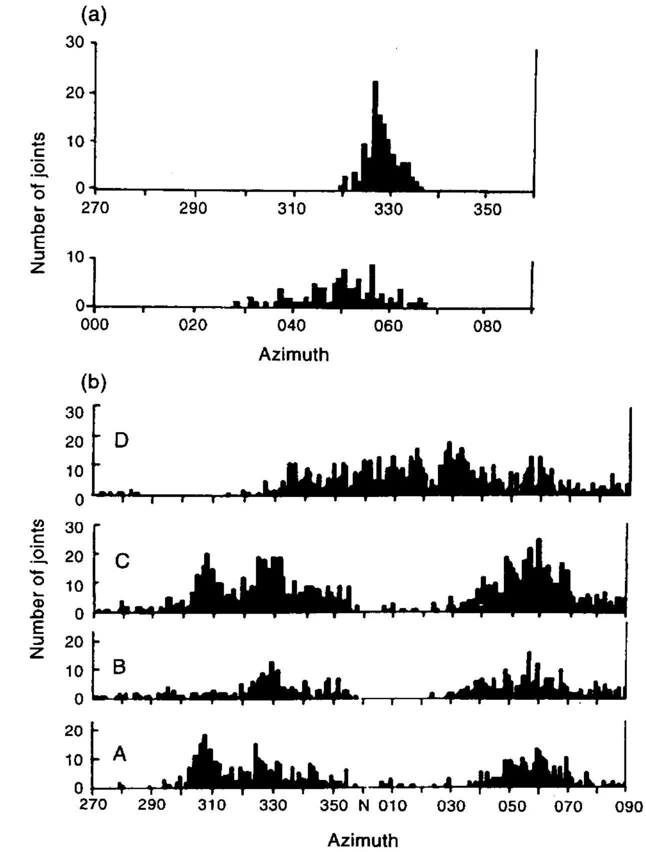



Fig. 2. Histograms of regional joint orientation in Eocene chalk around Beer Sheva (after Bahat and Grossman, 1988). (a) Strike orientation from a single outcrop in the Lower Eocene (Bahat and Grossman, 1988, station 20). (b) Strike orientation from 63 outcrops. A and B, Lower Eocene in the southern part of the Shefela syncline and the northern part of the Beer Sheva syncline, respectively; C, combined A and B; and D, Middle Eocene in the northern part of the Beer Sheva syncline.

Fig. 3. A fracture cutting Lower Eocene chalk in the Shefela syncline. En échelon fringes occur on both the upper and lower sides of the parent joint. There is a clockwise rotation of the en échelon segments with respect to the parent joint in the upper fringe and an counterclockwise rotation in the lower one. Cracks in the two fringes differ considerably in shape. Scale bar is 10 cm.

massive chert strike 342° ± 5° (this outcrop is inaccessible for a detailed study).

*Station W. S. (0595/1283)*

A composite fracture consisting of several subparallel joints interacting with each other on a common plane occurs on a vertical exposure of a 1.9-m thick chalk bed in Wadi Sécher (W. S.). The fracture morphology of the composite fracture indicates a multi-stage growth (Fig. 5a). This fracture surface strikes 073°, and is divided into five parts (Fig. 5b). Fracture initiated at the upper middle area of part A, or perhaps in the eroded upper left-hand side of this part, as revealed by two partly eroded concentric undulations. First, propagation was sideways and downward, ultimately arresting at a subhorizontal 'first fringe' of small en échelon cracks. Upward propagation was limited, probably by the layer boundary (which is eroded). Part B represents a second stage of growth, and is manifested by a fringe of en échelon cracks which initiates below the first fringe. Part C shows a new joint initiation in contact with the edge of en échelon segments from part B (resembling an observation made by Helgeson and Aydin, 1991). The plume of the new joint propagated bilaterally and downward. It arrested below at an elliptical undulation, but continued to propagate on the distal side. At the lower boundary there is a transition into en échelon cracks in a fringe that surrounds elliptically the parent joint C.

All en échelon segmentations in this outcrop are rotated clockwise with respect to the composite joint surface, except part D which presents a special case (at the centre of Fig. 5). Part D is characterized by large en échelon segments, and each one is marked by an individual plume. Strike measurements on the upper and lower parts of the segments show that the initial breakdown below the 'first fringe' was clockwise (from 073° to 080–083°). However, the downward propagation of the segments below the undulations was combined with some 7–8° counterclockwise azimuth rotation (e.g. from 083° to 076°, Fig. 5b). Each plume is cut laterally by an irregular torn surface (step) which breaks the segment subnormally on its right side. The torn surfaces do not have distinct fracture surface morphologies. Two adjacent flat en échelon surfaces combine again at the lower edge of one of the stepped surfaces, and plumes from the two surfaces unite below this edge and propagate downward (Fig. 5). This tear surface of finite dimensions is less common than other tear surfaces which end in free surfaces (at the layer boundaries) and do not have closed ends; revealing the complicated nature of the subsequent steps between the en échelon surfaces (Cooke and Pollard, 1996). Part E is a lateral continuation of part D, but it is largely eroded. The two parts are separated by a late joint that cuts the composite fracture at right angles.

Undulations (rib marks) form on the fracture surface by mixed modes I and II (e.g. Lawn and Wilshaw, 1975).

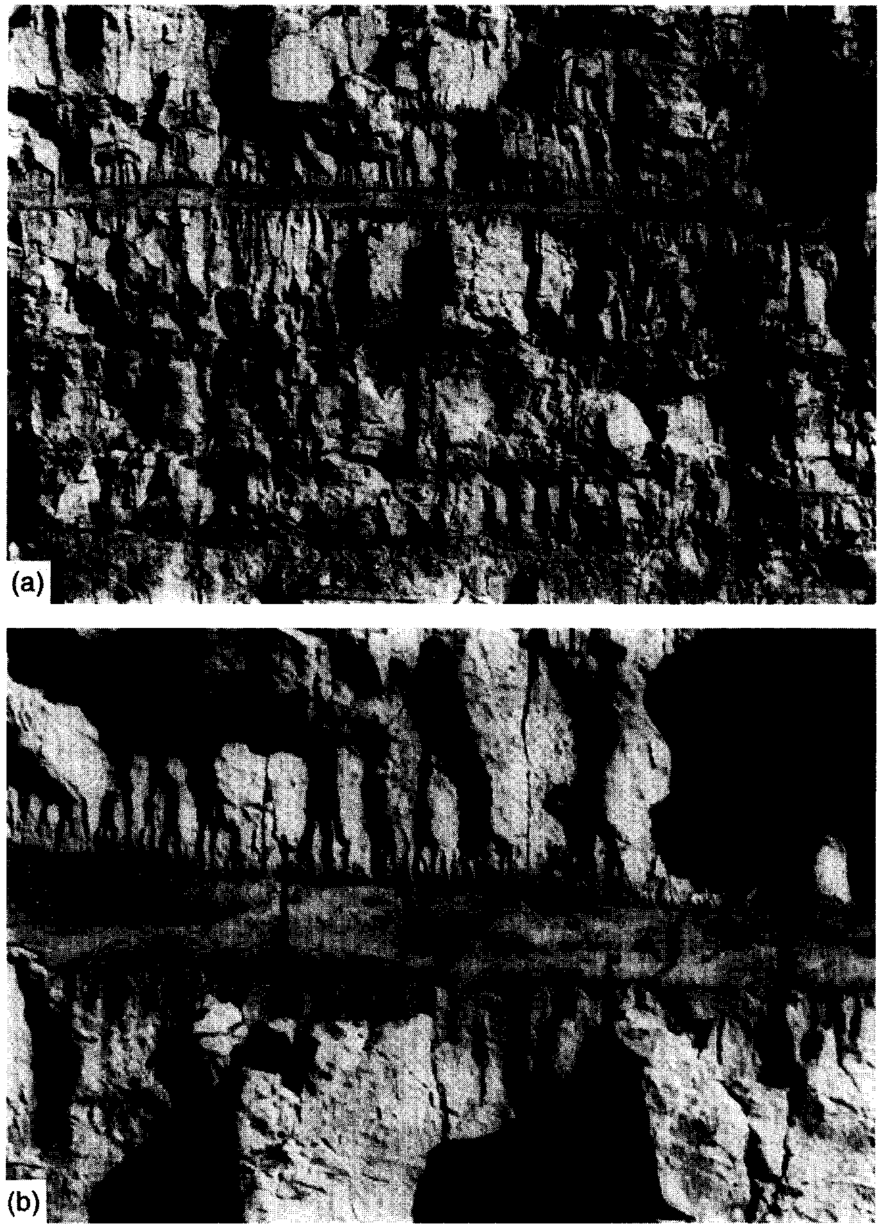



Fig. 4. Alternating chalk layers and chert beds in the Mor Formation near Beer Sheva. (a) En échelon segmentation occurs in chalk only along contacts with joints cutting massive chert beds, but not along contacts with beds of chert nodules. Scale in the lower part of picture is 1 m. (b) Enlargement of part of (a). Note the 'root zone' of several small segments associated with each larger one.

Often they appear synchronously with plumes, and on such occasions rib marks and barbs cross each other orthogonally. Several dark subhorizontal undulations continue through parts B, D and E, and the rib marks and barbs cross each other orthogonally on segments of part D.

*Station Y (0616/1283)*

In station Y from Wadi Naim the typical pattern of tens of fracture markings (Bahat, 1991a, p. 224) is that of a single fringe below the parent joint, characterized by a clockwise rotation of the en échelon segments with respect to the parent joint. The same rotation occurs in discontinuous and continuous breakdowns. Discontinuous (abrupt) breakdowns are more common than continuous (gradual) breakdowns, but both styles occur in the same bed. Discontinuous breakdowns initiate close to the layer boundary (Fig. 6a); whereas continuous breakdowns of the joint to en échelon segments generally initiate between the layer boundaries, somewhere close to the centre of the joint (Fig. 6b).

Generally the vertical parent joints are planar, and azimuths along the strike vary within $\pm 2^{\circ}$. However, the 'curved joint' undulates considerably along the strike, from 029° to 046°, while the azimuths of the en échelon segments maintain a much lower spread, from 043° to 047° (Fig. 6c). The twist angle is determined by the

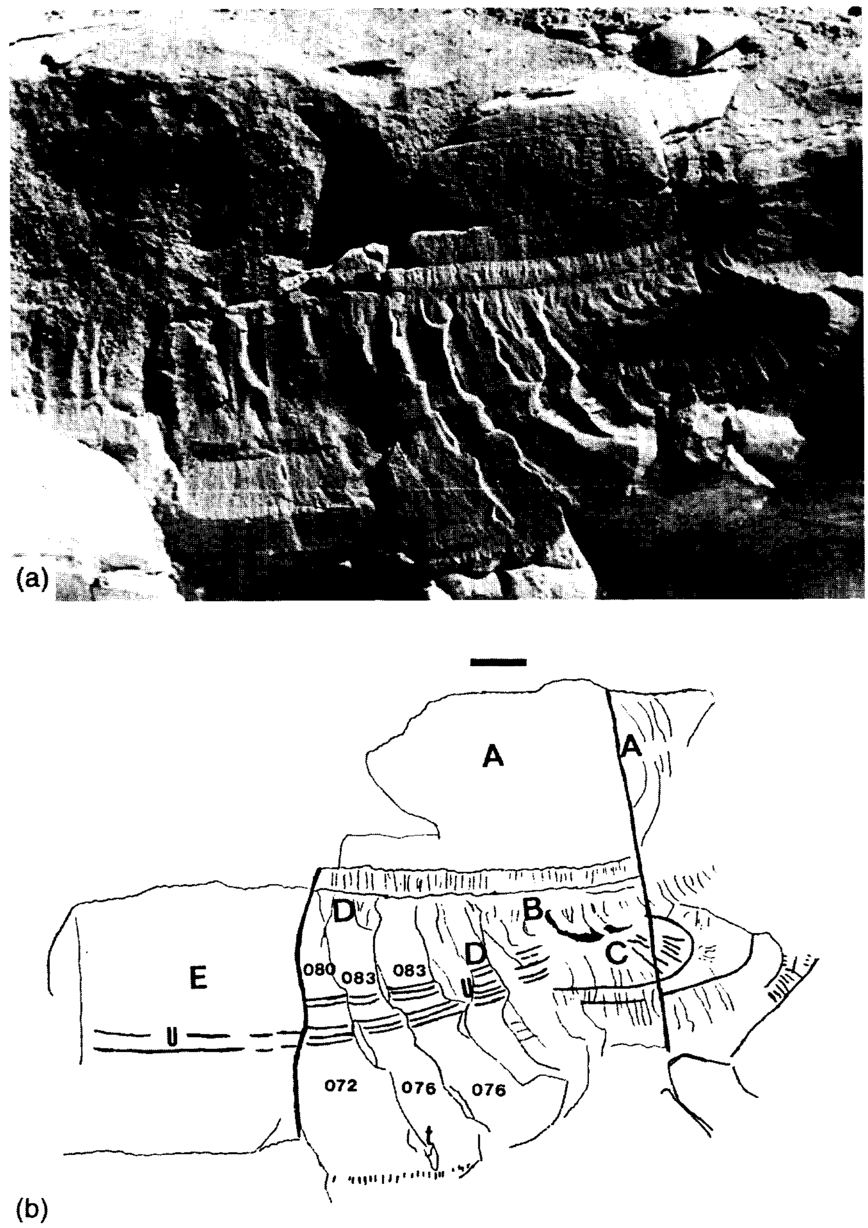
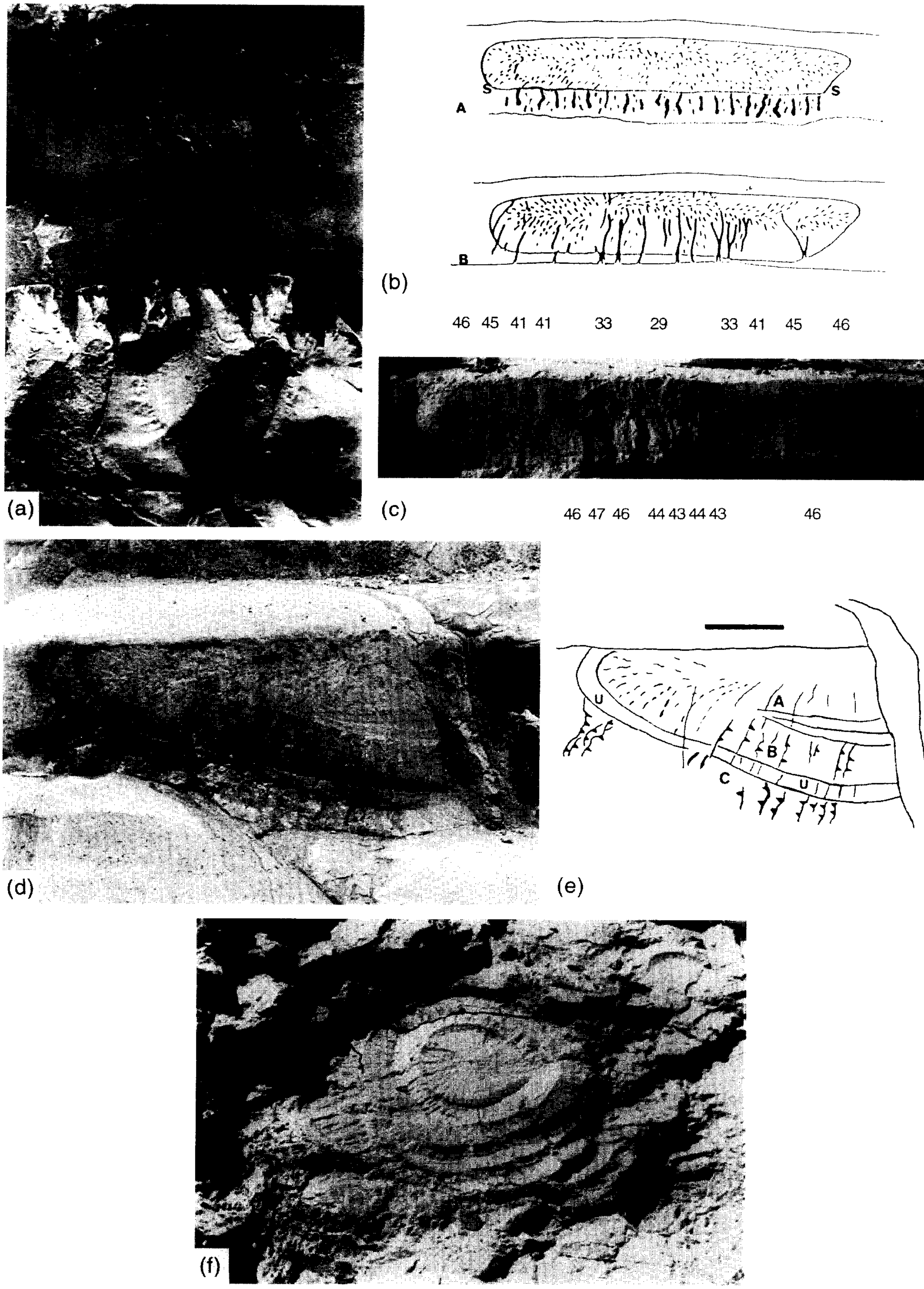



Fig. 5. (a) A composite joint in Wadi Sécher. (b) This fracture surface is divided into five parts, from A to E, see the explanation in the text. Locations of the lower edge of the finite tear surface and dark undulations are marked by t and u, respectively. Numbers on segments are azimuth degrees. Heavy subvertical lines cutting parts of A, B and C, and separating E from D, are late fractures. Scale bar is 15 cm.

difference between the strike of the parent joint and the en échelon segment at the same location. Evidently, the twist angle shows a pronounced range on the curved joint, from 0° to about 15°.

Previous observations along Wadi Naim have shown a typical growth sequence from an early 'embryonic' circular crack in the chalk to a larger elliptical fracture and then to a bilateral joint, whose extents are given by the bilateral plume on its surface (Bahat, 1991a, pp. 157 and 212). In the curved joint, segmentation initiated discontinuously from the lower, early elliptical boundary at about the centre and became continuous towards the two distal ends (Fig. 6c). The twist angle is maximum underneath the elliptical boundary, ranging from 10° to 15°, and it is reduced to 0–1° close to the distal ends of the joint (although measurements on the left distal side are difficult to obtain, the subparallelism between the joint and segments suggests small twist angles). There is a greater number of segments along a given distance (parallel to the strike) underneath the elliptical perimeter compared to other regions on the joint surface.

The 'concentric joint' is divided by delicate concentric undulations into several concentric zones which show different behaviour of the plume (Fig. 6d). The sense of rotation of overlapping barbs is counterclockwise in zones A and B (Fig. 6e). In the fringe (zone C) there is a mixed stepping, both counterclockwise and clockwise rotations of en échelon cracks occur in close proximity,


(a)
S
S
A
B
(b)
46 45 41 41 33 29 33 41 45 46
(c)
46 47 46 44 43 44 43 46
(d)
U
A
B
C
U
(e)
(f)

so that the en échelon cracks with clockwise rotation conform with the general stepping in outcrop Y, but the counterclockwise rotations of the barbs and the second group of en échelon cracks do not.

Segmentation also occurs in the concentric joint along the undulation between zone B and the en échelon fringe (U in Fig. 6e). However, there is no overlapping between neighbouring segments. Discoid joints, which cut Cenomanian chalk on Mt Carmel in the northern part of Israel, display a non-uniform segmentation in a series of concentric fringes. No segment overlapping occurs in the lower and upper parts of the discoid (Fig. 6f), resembling a similar feature along the undulation on the concentric joint (Fig. 6d).

*Station R (0617/1284)*

A large fracture marking occurs on a vertical joint cutting a layer 1.6 m thick. This layer rests on floor R (Bahat, 1991a, p. 247) and its upper boundary contains a thin bed of gypsum with small crystals oriented perpendicular to the bed (Issar *et al.*, 1988). Joint propagation was initiated close to the upper boundary of the layer and mostly propagated sideways and downwards. The joint surface consists of an upper, concave-down, semi-ellipse, which is separated by a curved shoulder from a fringe of en échelon segments in the lower part of the joint (Fig. 7).

The semi-ellipse is subdivided into three concentric parts. A is a delicate asymmetric bilateral plume showing the location of fracture initiation which characterizes the upper (and smaller) ellipse which is slightly suppressed at the upper layer boundary. The second ellipse, B, is also somewhat suppressed at the upper layer boundary, and is separated by a delicate boundary from the upper ellipse. Ellipse B is distiguished by the radial barbs (branches of the plume) which are in a partial continuation with the plume in A. The sense of rotation of these barbs is counterclockwise (their sense of rotation is determined like that of en échelon segments by the sense of their partial overlapping). The third part, C, is a semi-ellipse significantly truncated by the upper layer boundary, and is marked again by sporadic radial barbs which partly overlap and maintain a counterclockwise sense of rotation. The delicate but well-defined boundaries between parts A and B, and between B and C, reflect distinct fronts of the plume propagation.

A wide fringe in the lower part of the joint is segmented into coarse, large, en échelon cracks below the curved shoulder. The segments close to the centre of the curved shoulder reach the layer boundary, whereas away from the centre they seem to arrest before reaching the boundary. It has been observed that large fringes often grow around curved shoulders (Bankwitz, 1965). Boundaries of individual en échelon cracks are well defined, and there are no groups of small segments associated with the larger ones along the 'root zone' (Bankwitz, 1966). The sense of rotation of the coarse segments in the fringe is clockwise, compared to counterclockwise rotation of the delicate barbs in the early ellipses (parts B and C).

The fractures in outcrops W. S. and R cut relatively thick beds in contrast to fractures in the other outcrops of the present investigation. The former fractures initiate close to the upper boundaries of the layers they cut. Lack of close boundary constraints in the lower parts of these thick layers enabled fracture propagation downward in curved fronts (Bahat, 1991a, p. 157). These are shown by downward convexing undulations (Fig. 5) and by the arched shoulder shown in Fig. 7.

*Outcrop X (0620/1289)*

Fringes of dilatant en échelon cracks both above and below the parent joints (Fig. 8) are three-dimensionally exposed in outcrop X on a quarried face, as well as on an adjacent platform. The sense of rotation of the segments in the two fringes is counterclockwise. This is an opposite rotation with respect to the one in outcrop Y, although these two outcrops are geographically fairly close to each other (Bahat, 1986).

*The trench-outcrop T (6201/1256)*

A parent joint which undulates along the strike from 62° to 64° has above it a fringe of mixed steppings. The fringe is subdivided into three distinct parts. Five en échelon segments rotate counterclockwise and vary in

Fig. 6. Fracture morphologies from outcrop Y. (a) Part of a joint with a discontinuous breakdown along its lower part (Bahat, 1991a, p. 225). Both the plume in the upper part of the parent joint and the en échelon segments in the fringe are rotated clockwise. (b) Hypothetical two segmentation styles of parent joints marked by bilateral plumes (dashed lines). A, discontinuous breakdown into en échelon cracks (solid lines) along a shoulder S below the main plume, secondary plumes may develop on the segments; B, continuous breakdown into en échelon cracks which initiates on the parent joint at various locations along the main plume. (c) The curved joint with a discontinuous breakdown below the elliptical perimeter of the early 'embryonic' joint at the centre and a continuous breakdown at the distal sides. The joint undulates along the strike. The upper numbers are measured azimuth degrees (from 029° to 046°) on the parent joint. The lower numbers are azimuth degrees (from 043° to 047°) on en échelon segments. The delicate barbs on the joint surface represent a bilateral plume. Scale bar is 50 cm. (d) A joint initiated at a boundary of a previous joint on the right and propagated towards the left, as shown by concentric undulations and radial barbs (after Bahat, 1991a, p. 158). (e) A drawing of (d). A discontinuous breakdown occurs below the outer undulation U. The sense of rotation of overlapping barbs is counterclockwise in zones A and B. Segments on the right-hand side of the fringe rotate clockwise with respect to the parent joint (triangles along the heavy lines point to the direction of overlapping segments). This rotation conforms with the other rotations in outcrop Y. However, segments on the left-hand side of fringe show counterclockwise rotation, which is in conformity with the rotation of overlapping barbs on the joint surface. Scale bar is 20 cm. (f) A discoid joint from Mt Carmel in northern Israel cutting Cenomanian chalk. The discoid displays several concentric fringes with a non-uniform breakdown. Segment rotation is counterclockwise on the left-hand side of the discoid (the shadow is on the left-hand side of overlapping segment) and is clockwise on the right-hand side of the joint (where the shadow is on the right-hand side of overlapping segment). There is no overlapping in the upper and lower parts of the discoid.

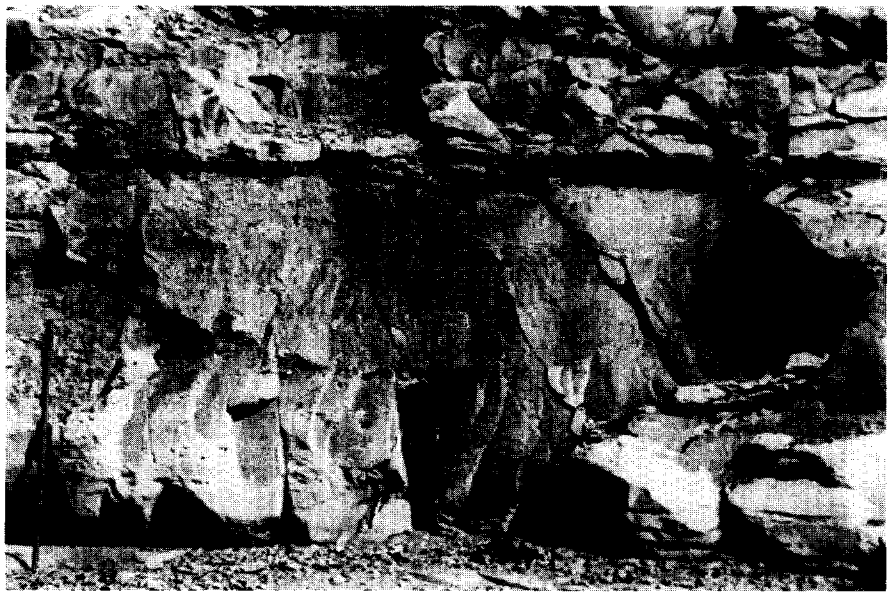


Fig. 7. A joint from outcrop R showing a division into two parts. An upper semi-ellipse parent joint, which is concave downward, is separated by a curved shoulder from a fringe of en échelon segments below. There is a (hardly visible) division of the parent joint into three concentric zones (see text for more information). Scale is 1 m.

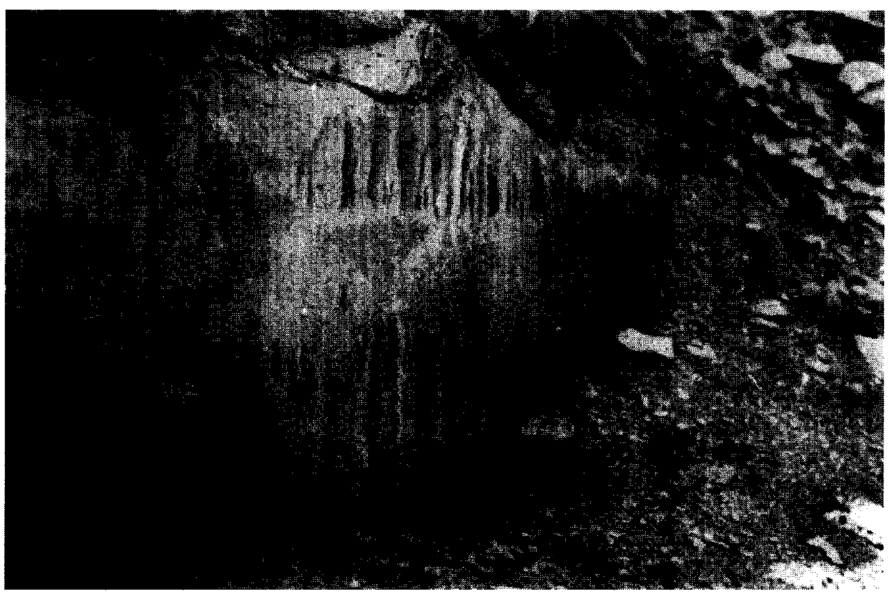


Fig. 8. Fringes of dilatant en échelon cracks both above and below the parent joints in outcrop X. The sense of rotation of the segments in the two fringes is counterclockwise. Note the similarity in shape and size of segments in both fringes. Scale is 15 cm (after Bahat, 1986).

orientation from 047° to 049° in the part on the left-hand side, and three segments rotate clockwise and range from 073° to 080° in the part on the right-hand side. In the central part, between the two, there are nine segments that rotate counterclockwise (Fig. 9). These segments are, however, smaller than the segments from the other two groups and their azimuths are quite difficult to measure, but their strikes appear to be close to those of the other segments on the left-hand side. The segments of the central part overlap the segments on the left (both show a counterclockwise rotation), so that the small segments appear to have followed in sequence the larger ones. The two small segments on the extreme right of the central part seem to overlap the left segment of the group which rotates clockwise; again, implying that the large clockwise rotating segments preceded the small ones.

*Orientation of en échelon segments in three directions*

The orientation and several key properties of en échelon segments from various joint fringes in the Beer Sheva syncline are summarized in Table 1. Three sets of

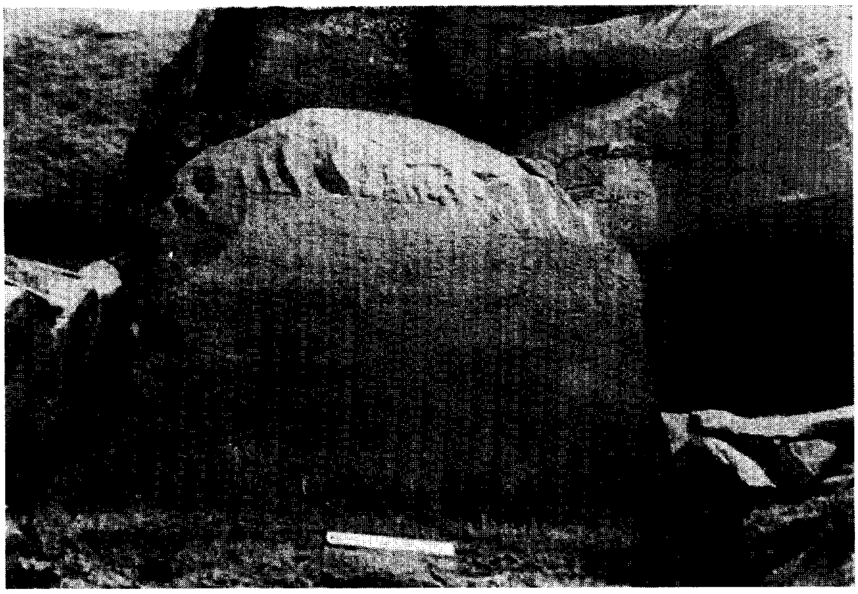


Fig. 9. A parent joint which undulates along the strike from 62° to 64° with a fringe that is subdivided into three distinct parts. The en échelon segments vary in orientation from 47° to 49° in the part on the left-hand side, and from 73° to 84° in the part on the right-hand side. In the central part, between the two, the orientation of segments varies from 74° to 80°. The sense of rotation of segments is counterclockwise in the two parts on the left side, and is clockwise in the part on the right-hand side of fringe. Scale is 15 cm.

en échelon cracks come out when grouping together the segments of overlapping orientations: (a) segmentation in the range 338–342° which rotates counterclockwise with respect to the joints that strike from 350° to 014° at station X; (b) segments in the 043–047° range that rotate clockwise with respect to the set 029–046° and counterclockwise in relation to the set 062–064° from stations Y and T, respectively; and (c) the range 072–083° which rotates clockwise relative to the sets which strike 073° and 062–064° in stations W. S. and T, respectively. It appears that the sequence of superposition of the three groups of segments in the fringe shown in station T (Fig. 9) was: first, the clockwise segments, striking from 073° to 080°; then the large counterclockwise segments striking from 047° to 049°; and the last to form were the small counterclockwise-rotating en échelon cracks at the centre.

### *The relationship of en échelon crack length to layer thickness and layer-parallel slip*

Lengths of en échelon segments cutting beds of various thicknesses were measured at stations Y, R and Q in Wadi Naim. At these locations en échelon fringes occur only below the parent joint (Bahat, 1986). The length of the longest segment for each joint was measured from its initiation point to the lower layer boundary. The layer thickness at that location was measured as well (estimated error for both measurements is $\pm 2$ cm). Five continuous and 14 discontinuous segmentations were studied and the results are summarized in Fig. 10.

There is a positive linear fit ($R^2 = 0.928$) of the readings for discontinuous segmentation which is given by

$$l = 0.270T + 0.829,$$

Table 1. Orientation of joints and associated en échelon segments in the Beer Sheva syncline and corresponding compression*

| Station | Name of joint/set | Azimuths of joint/s (°) | Azimuths of segments (°) | Segmentation: above joint | Segmentation: below joint | Rotation of segments: clockwise | Rotation of segments: counterclockwise | Corresponding compression |
|---|---|---|---|---|---|---|---|---|
| (1) W. S. | Part D | 073 | 073–083 | | + | + | + | ? |
| (2) Y | The curved joint | 029–046 | 043–047 | | + | + | | Local |
| (3) Y† | Set O-Y3 | 012–035 | 026–053 | | + | + | | Local |
| (4) X‡ | Set X | 350–014 | 338–342 | + | + | | + | N–S, Regional§ |
| (5) T | Mixed stepping | 062–064 | 047–049 | + | | | + | Local |
| | | | 073–080 | + | | + | | ? |

*Stations correspond to those in Fig. 1. All results are from vertical outcrops.
†From Bahat (1986, table 4).
‡From Bahat (1986, table 1).
§Southern part of the Dead sea rift.

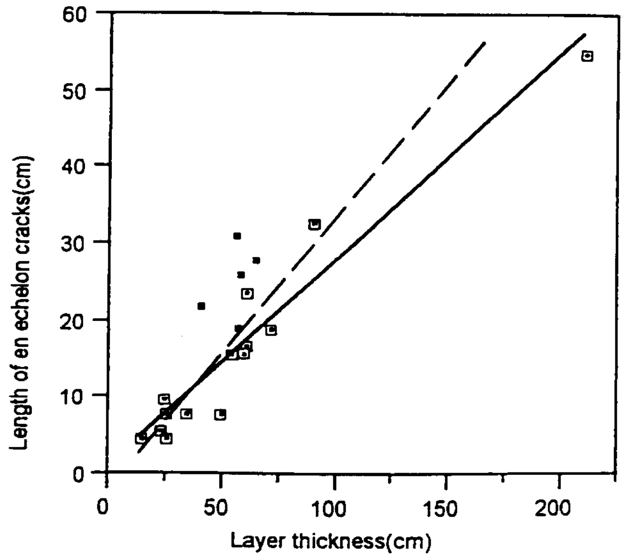



Fig. 10. A plot of layer thickness vs length of en échelon cracks in chalk outcrops along Wadi Naim. The 14 large open squares and the five solid squares designate, respectively, discontinuous and continuous breakdowns. The solid and and dashed plots are the products of 14 and 13 readings of discontinuous breakdowns, respectively. The last reading ($T = 210$ cm, $l = 55$ cm) is omitted from the latter. The former is considered to be a better representation of the results.

where $l$ is segment length and $T$ is bed thickness. $R^2$ is reduced to 0.836 when one reading ($T = 210$ cm, $l = 55$ cm) is omitted. The readings for the continuous segmentations scatter above the curve.

There is no evidence of any layer-parallel slip along either chalk–chalk or chalk–chert boundaries linked to en échelon fringes in the stations investigated in this study. There are, on the other hand, many inter-layer radial markings on horizontal layer boundaries which reflect tensile vertical strains in the chalk (Bahat, 1991a, p. 298).

## THE RELATIONSHIPS BETWEEN PARENT JOINTS AND EN ÉCHELON SEGMENTS

### *Twisted discontinuous vs curved continuous breakdowns to en échelon segmentation*

Two distict fracture propagation paths may be formed under mixed mode I + III loading. An abrupt loading produces abruptly twisting en échelon planar cracks. On the other hand, if mixed mode loading is increased as the crack propagates, the resulting path follows a curved trajectory (Cooke and Pollard, 1996). These two paths are manifested by part D of the composite fracture in station W. S. (Fig. 5). The clockwise breakdown occurred in part D into a path of twisted segments (azimuths 080–083°). As the loading increased, while these segments propagated downward beyond the undulations, a curved path took over and a counterclockwise rotation of the segments occurred (into azimuths 072–076°).

It may be inferred from the experimental results by Cooke and Pollard (1996) that, generally, breakdowns which are manifested by en échelon segments formed by abrupt twisting reflect discontinuous processes associated with abrupt changes in I + III loadings between the development of the parent joint and the fringe (e.g. Fig. 6a). On the other hand, gradual breakdowns imply continuous processes associated with gradual twisting, responding to mixed mode loading as the joint propagates and segmentation occurs (Fig. 6c). The gradual mixed mode loading probably does not involve any time-break during the process. It is, however, possible that time-breaks do occur during discontinuous processes, but their extents are not known.

In outcrop Y, continuous and discontinuous breakdowns occur in the same bed. Furthermore, the curved joint was first segmented by a discontinuous process underneath the early elliptical fracture on the curved joint, and the second breakdown was continuous along the distal edges (Fig. 6b & c). The proximity of these two breakdown styles suggests that their fracture conditions were not drastically different.

The curved joint (Fig. 6c) and joint W. S. (Fig. 5a & b) demonstrate that the continuous breakdown may have two manifestations. First, a gradual growth of en échelon segments from distal locations on the parent joint (Fig. 6c) and, second, a gradual curving of growing segments (Fig. 5). These two cases are associated with relatively small twist angles (0–11°) compared to the discontinuous breakdown in the curved joint, which occurs in larger twist angles, and to the discontinuous breakdowns in various joints from stations Y and R, which also occur in larger twist angles (generally from 12° to 25°).

### *Opposite senses of segment rotation in a single fringe*

The observation of the close proximity of both clockwise and counterclockwise rotations of en échelon cracks in a single fringe is quite intriguing. Based on experimental results (Cooke and Pollard, 1996) it follows that the clockwise twisted segments in Figs 6 & 9 reflect right-lateral shear, and the counterclockwise twists disclose left-lateral shear. There is a need to distinguish between two cases of opposite senses of segment rotation in a single fringe: first, in a curved fringe, and second, in a straight fringe.

Opposite senses of segment rotation in curved fringes are quite common in fractured engineering materials and in association with discoid joints (Fig. 11). Segment rotation is counterclockwise on the left-hand side of a rounded parent fracture in steel and is clockwise on the right-hand side of the fracture (Fig. 11a, after Yukawa *et al.*, 1969). Exactly the same pattern occurs around a discoid joint in the Frauenbach Quarzite from the Thuringian Schiefergebirge (Fig. 11b, after Bankwitz and Bankwitz, 1984). In fact, Bankwitz and Bankwitz (1984, fig. 2) observed the symmetry created by the reverse segmentation (compare with Fig. 6f). The same segmentation pattern was repeated on the surface of a fractured glass rod

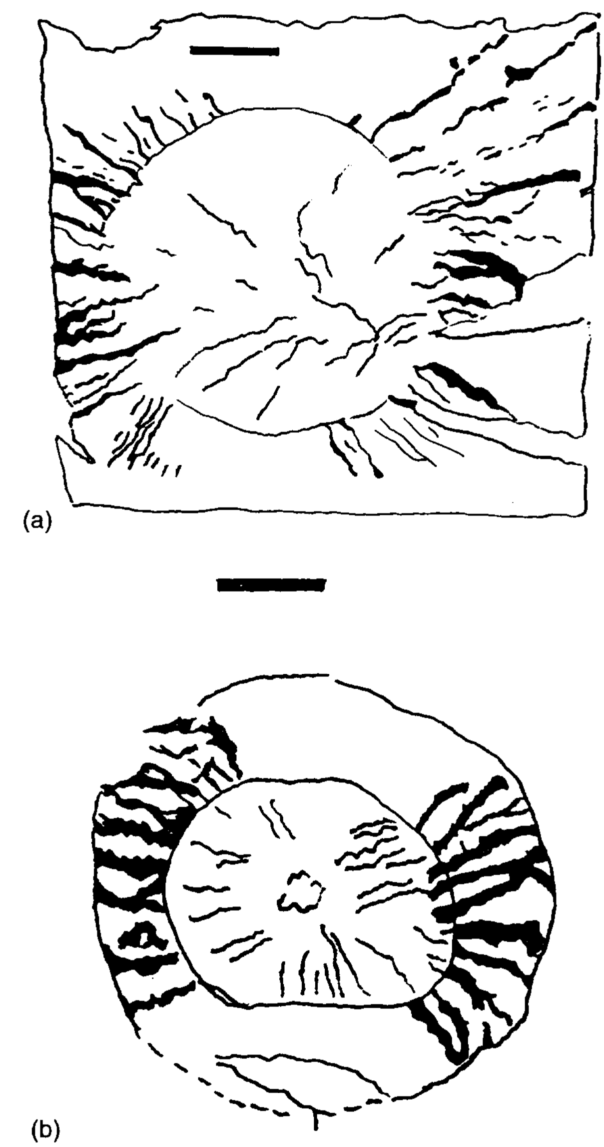


Fig. 11. Drawings of fringes around circular parent fractures. (a) Segment rotation is counterclockwise on the left side of a fracture in steel and is clockwise on the right side of the fracture (after Yukawa *et al.*, 1969, fig. 1). Scale bar is 12 mm. (b) A repeated pattern from (a) around a discoid joint in the Frauenbach Quarzite from the Thuringian Schiefergebirge (after Bankwitz and Bankwitz, 1984, fig. 3). Scale bar is 5 mm.

which failed under the combined operation of modes I and III (Sommer, 1969, fig. 2).

This fracture pattern is a result of a slight deviation of the tensile axis (by not more than several degrees, Sommer, 1969) from normality to the parent fracture. It caused two segmentation operations advancing in opposite directions (Fig. 11), rather than a single propagation that would surround the entire boundary of the circular parent fracture. Such a single propagation would occur under 'pure tension', which is probably difficult to achieve, and therefore would probably be less common. The latter scenario is shown by the discoid from the Carmel (Fig. 6f) where there is no segment overlap in certain parts of the concentric fringes. This is assigned to 'pure tension' which was applied on the central part of the discoid but not on its right and left parts. It suggests that even locally the stress field was not homogeneous.

The 'concentric joint' is a case in point of opposite senses of segment rotation in a single curved fringe (Fig. 6d & e). This phenomenon is analogous to the opposite segment rotations in the curved fringes of the fractured engineering materials and the discoid joints mentioned above. This can best be assigned to local deviations of the minimum principal stress direction from perpendicularity to the parent joint, because similar joints adjacent to the concentric joint do not show opposite segment rotation. A remote deviation of the minimum principal direction would result in similar segment rotations in neighbouring joints.

Quite a distinct situation is opposite segment rotation in a single straight fringe. The opposite rotation shown at station T (Fig. 9) cannot be explained by analogy with fracture in the discoid joints. This reverse rotation seems to reflect changes in remote stresses on relatively small scales (kilometres in the Beer Sheva syncline), and it may be compared with the reverse senses of segment rotation in the Appalachian Plateau (Younes and Engelder, 1995). Younes and Engelder (1995) observed that the opposite rotation of en échelon segments along joint fringes on large scales recorded the sense of palaeostress rotation, which they then related to the bending of the Appalachian Plateau during the Alleghanian orogeny. The correlation of segment rotation at station T to remote stresses is supported by a further interpretation of the data in Table 1.

This hypothesis would be challenged if a straight fringe similar to that found at station T were to show a transformation from a right-lateral shear to a left-lateral shear through a transitional zone of 'pure tension', as in Fig. 6(f).

*Similar fringes below and above the parent joint*

En échelon segments maintain counterclockwise rotation in fringes close to both the lower and upper boundaries in outcrop X (Fig. 8), and the two fringes, above and below the parent joint, have similar geometries and shapes. This coupling implies that en échelon segmentation occurred simultaneously in the two fringes, which happened following the required diagenetic process for the equalization of the lithological properties along both the lower and upper layer boundaries. The fractographic features of the joints in outcrop X are quite distinct from the fractographies of all other joints in the investigated area.

*Opposite rotations of en échelon segments in fringes below and above the parent joint*

Significant dissimilarities in size and shape of the fringes would reflect a deviation from simultaneous en échelon segmentation. The two fringes in Fig. 3 differ in the directions of segment rotation and in segment size and geometry, quite possibly suggesting that the two fringes responded to different loadings.

It appears that the fracture sequence in Fig. 3 was: (a) propagation of the parent joint from left to right, as shown by the plume; (b) segmentation of the lower fringe in a rational continuation with the plume on the parent joint and following a change in the loading direction; and (c) segmentation of the upper fringe.

*Timing relationships of plumes, undulations and en échelon segmentation*

The contact relationships of rib marks and barbs, combined with their interactions with crack segmentation, are useful in estimating changes in local stresses and they enable different stages in the fracture process to be distinguished. The plume on the curved joint (Fig. 6c) is bilateral and the segmentation shows a uniform clockwise rotation which corresponds to one direction of the plume but not to the other. The implication is that breakdown occurred after the completion of the growth of the bilateral joint. Hence, the growth of the curved joint was in several stages: (a) fracture into an early elliptical joint; (b) growth along the upper part of the layer into a bilateral joint which curves along the strike; and (c) breakdown of the joint into segments that propagated downwards.

What was the growth rhythm of the various segments along the curved joint? There seem to be conflicting observations. On the one hand, the breakdown into segments along the curved joint occurred under various $K_{III}/K_I$ ratio conditions (see below); which may imply that segmentation was not synchronous along the joint, but it occurred at the centre before developing at the distal ends. On the other hand, it seems quite likely that the various segments reached the lower part of the joint simultaneously, as demonstrated by a series of undulations that mark all the segments at the same height and practically appear to be a monotonous morphological feature (Fig. 6c). The possibility of a non-synchronous segment initiation, but a synchronous completion of segmentation, is not ruled out.

Some joint surfaces show divisions into concentric zones which are bounded by undulations (Figs 5, 6d & 7). These undulations not only represent the fracture front at 'rest' but they also mark the loci of stress redistribution (Murgatroyd, 1942). This is revealed by changes in the sense of rotation of overlapping barbs with respect to the en échelon segments (Figs 6e & 7) or by changes in the segment azimuths (Fig. 5, part D).

Undulations can be useful in the interpretation of the inter-relationships of individual joints associated in a common composite fracture. The series of subhorizontal undulations which continue through parts B, D and E (Fig. 5) suggest an early synchronization of the downward propagation of these parts.

*The spread of azimuths in joint sets vs en échelon fringes*

The spread of azimuths in individual joints and in joint sets is often greater than the spread of azimuths of the en échelon cracks associated with them. One example is shown in Fig. 6(c). Other joints in outcrop Y show similar patterns. A summary of fracture orientations in outcrop X shows that the mean azimuth of the joints is N03°W with a standard deviation of $\pm 12°$ (63 measurements). The corresponding result for the en échelon cracks along the fringes of the joints is N23°W $\pm 6°$ (91 measurements) (Bahat, 1986). The usefulness of this observation is described below in connection with palaeostress estimation.

*The influence of lithological interfaces on en échelon segmentation*

The en échelon segmentation in chalk beds of the Lower Eocene occurs exclusively along welded interfaces with massive chert (Fig. 4). Segmentation in the chalk occurs along lithological interfaces near joints in the chert beds. Therefore, jointing in the chert beds preceded segmentation in the chalk. This association resembles a situation where en échelon cracks in shale are found in contact with joints in siltstone in the Appalachian Valley and Ridge (Engelder *et al.*, 1993, fig. 15-10). Engelder *et al.* (1993) consider that this fracture association implies sequential processes, where jointing preceded segmentation.

An intriguing observation was made by Roberts (1995, fig. 4d), who showed a joint marked by a plume which cut a thin limestone bed (about 10 cm thick). This plume uninterruptedly transformed into en échelon segmentation in the interbedded shale layer, while both the plume and the segments maintained counterclockwise rotation with respect to the joint.

Common to these three examples is the transition from a joint in a competent rock to en échelon segmentation in an adjacent welded less competent rock. It appears that stress along the welded contact of the two rocks resulted in a mode III shear which led to segmentation. It may also be speculated that a slight tendency for segmentation which existed in the less competent layers (see the mechanism of fracture slanting below) was suppressed (e.g. by overburden) and the induced mode III shear along the welded contact overcame the suppression.

## EFFECTS OF LOCAL STRESSES VS REMOTE STRESSES ON EN ÉCHELON SEGMENTATION IN THE EOCENE CHALK

*An ascending order of fracture morphologies on joint surfaces*

It is useful to set the various fracture morphologies on joint surfaces in an ascending order of compounded elements. These can be correlated with various stress scenarios, from 1 to 12 (Fig. 12). In a crude way, scenarios 1–7 represent various manifestations of fracture morphologies that respond to changes in local stresses, whereas scenarios 8–12 show fractographies that may help to evaluate changes in directions of remote stresses. A fuller interpretation of this concept is given in the next sections under the headings 'Local stresses' and 'Remote stresses'.

Scenarios 1–3 represent, respectively, unilateral, bilateral and concentric modes of joint propagation. Common to these morphologies is the perpendicularity of the minimum principal stress with respect to the parent joint, with possible slight deviations from it (Bahat, 1987). Local constraints dictate the choice among the three, and a prolonged fracture in a thick layer would favour the third one (scenarios 1 and 2 are well known, and scenario 3 is manifested by the parent joint shown in Fig. 7 before the breakdown).

Scenario 4 represents a composite fracture which developed by the merging of a series of subparallel smaller joints, propagating in the same approximate stress field (Fig. 5). Scenario 5 describes the propagation of one part of the composite fracture (Fig. 5, part D) which is the only joint which deflects in response to a slight deviation of the local minimum principal stress from perpendicularity. A deviation of the remote principal stress from perpendicularity to the composite fracture would have resulted in the deflection of additional parts of the composite fracture as well.

Scenario 6 results when the local perpendicular minimum principal stress rotates somewhat with respect

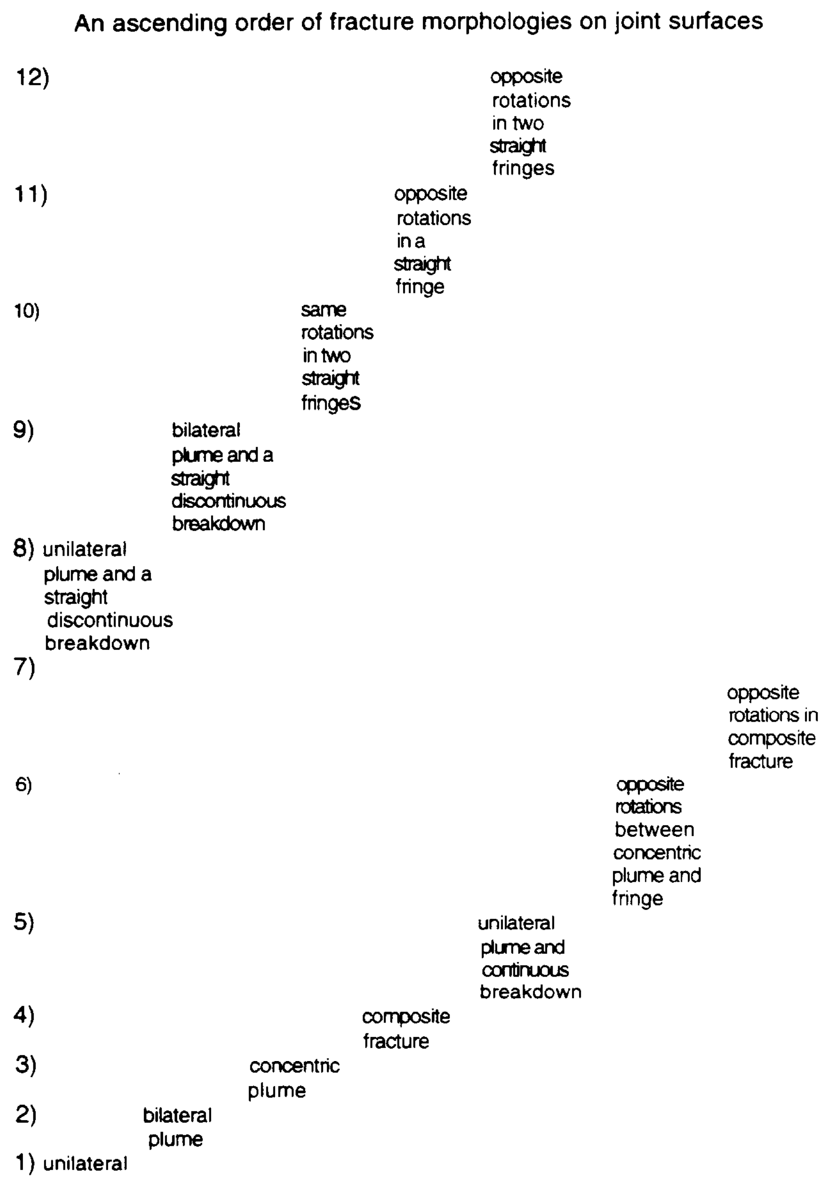



Fig. 12. Fracture morphologies on joint surfaces in an ascending order of compounded elements which correlate with 12 stress scenarios.

to the parent joint, so that a left-lateral shear changes to a right-lateral shear (Fig. 6e). It appears that a segmentation without overlapping (Fig. 6f) signifies a transitional stage between left-lateral and right-lateral shears. Scenario 7 concerns alternating shear among neighbouring joints that constitute a composite fracture. This is a situation known to the author from massive sandstones, but is not known in chalk.

Scenarios 8 and 9 are recognized by the abrupt transitions between the parent joint and the en échelon straight fringe. These structures have the necessary patterns and are good candidates for the identification of a remote stress directions. Scenario 10 (Fig. 8) may be considered as an 'upgrading' of scenarios 8 and 9, also leading to the identification of remote stress directions.

Scenarios 11 and 12 are related to opposite segment rotations in straight fringes (Figs 9 & 3, respectively). They also provide indications for the distinction between different remote stress directions.

### *Local stresses vs remote stresses*

It is useful to make the distinction between the influences of local and remote stresses on the formation of en échelon cracks.

*Local stresses*. Field observations indicate that much of the inter-relationship of the parent joint with the fringe, and the behaviour of the en échelon segments, is influenced by local conditions. Here are a few observations which categorize local influences.

(1) There is a correlation between the size of en échelon cracks formed by the discontinuous breakdown and layer thickness (Fig. 10).

(2) Adjacent continuous and discontinuous en échelon breakdown styles occur in the same rock layer. For instance, the various measurements of en échelon lengths vs the layer thicknesses in the thickness range 55–65 cm (Fig. 10) are all from layer 5 (Bahat, 1986, fig. 2), and they show both continuous and discontinuous breakdowns.

(3) Occasionally, timing relationships exist among plumes, the vertical en échelon cracks and the horizontal segments of undulations (Figs 5, 6e & 7). These relationships may occur on surfaces of individual joints or composite joints but not in association with any of their neighbours along the same layer.

(4) The sense of rotation of the segments in the fringe is clockwise, compared to counterclockwise rotation of the barbs on the parent joint (Figs 6e & 7). It seems highly unlikely that these two rotations reflect remote stresses. Quite possibly the counterclockwise rotation of the barbs is in response to local stress.

(5) The reverse segment rotation along the fringe of the concentric joint (Fig. 6e) reflects local conditions, rather than two distinct regional loadings.

*Remote stresses*. In spite of the important influence of local conditions on the behaviour of en échelon segments, there are certain parameters that can be quite clearly correlated with remote stresses.

(1) The decoupling of the fringe from the bilateral plume, which was described above (Fig. 6c), suggests that these two morphologies were formed in response to distinct stress conditions. Here, the preference of the plumes to propagate bilaterally rather than unilaterally demonstrates a response to local conditions. On the other hand, the azimuth uniformity of the en échelon segmentation in the fringe and other fringes of neighbouring joints in a set conforms with the influence of a remote stress.

(2) The decoupling of a fringe from a unilateral plume is as much an indication that the segmentation in the fringe represents a remote stress, as in the previous case when related to the bilateral plume. A good example is provided by Younes and Engelder (1995). The uniformity in the sense of rotation of en échelon cracks with respect to their parent joints over great distances is a manifestation of fracture induced by remote stress.

(3) The conformity of the clockwise rotation of en échelon segments with NNE joints at station Y and the strict counterclockwise rotation associated with the N–S joints at outcrop X suggest two sets responding to distinct remote loadings (Table 1).

(4) Specifically, fractures in outcrop X differ from all other fractures in the investigated area by the uniformity of their en échelon cracks in term of their sense of rotation, their size and shape, and in the fringes which occur both below and above the parent joint. This distinct assemblage of properties categorizes a distinct stress field.

(5) When a set of joints demonstrates a gradual azimuth rotation, the en échelon cracks at the fringes of these joints rotate in covariance with the joints (Bahat, 1986).

(6) Discontinuous breakdowns are distinct from continuous breakdowns by often having a series of small en échelon cracks along the shoulder distributed among the larger en échelon cracks. Such series of small cracks have been named the 'root zone' (Bankwitz, 1966). They generally maintain their azimuths in alignment with their larger neighbour en échelon cracks in response to remote stresses.

## THE STRUCTURAL INTERPRETATION OF EN ÉCHELON FRINGE CRACKS IN THE BEER SHEVA SYNCLINE

### *Three styles of en échelon segmentation in the Beer Sheva syncline*

Three styles of en échelon segmentation can be distinguished in the Beer Sheva syncline, which are represented by set X, set O-Y3 and the joint W. S., respectively. Set X has en échelon fringes both below and above the parent joint, and in both segment rotation is counterclockwise, ranging in strike between 338° and 342°.

Quite distinct from set X is set O-Y3 at station Y. Set O-Y3 has a fringe only below the parent joint and segment rotation is clockwise, ranging in strike between 026° and 053°. The curved joint at station Y undulates considerably along the strike but the segmentation is confined to a small azimuth range (043–047°), which is only a limited spread of the range 026–053°; pointing to their linkage being in the same general direction.

Joint W. S. is best represented by part D. The parent joint strikes 073° and the general clockwise segment rotation is in the 073–083° range. This is the same range and sense of rotation as measured on the fringe of the joint from station T, which strikes 062–064° (the significance of the occurrence of this latter fringe above the joint is not clear).

*Discrimination of joints into sets*

The discrimination of joints into sets may provide an important tool in determining palaeostress directions (Engelder and Geiser, 1980). However, this task is often quite difficult because systematic joints commonly occur in complex patterns (e.g. Younes and Engelder, 1995). Geologists often appreciate the difficulty in estimating a palaeostress direction because of the uncertainties involved in determining the spread of the joint population in a set, and the average azimuth of the set associated with this direction.

Particularly important is the observation mentioned above, that the spread of azimuths of en échelon segments in a set is often considerably smaller than the spread of azimuths of the parent joints associated with that set. The small azimuth spread of en échelon segments in a set enables quite readily a distinction to be made between neighbouring en échelon sets, and commensurately to discriminate between the respective joint sets associated with them.

This is well exemplified in Table 1. It would be difficult to distinguish between joint sets X and O-Y3 because of the overlapping ranges of fracture azimuths (350–014° and 012–035°, respectively). It is, however, considerably easier to distinguish between these sets on the basis of their distinct segmentation ranges (338–342° and 026–053°, respectively). Hence, the styles of en échelon segmentation and rotation can considerably improve confidence in the division of joints into sets and can facilitate the analysis of palaeostress directions.

*Palaeostress directions*

Set X is represented by the joints which strike between 350° and 014° in outcrop X. They seem to reflect a regional N–S post-Middle Eocene palaeocompression. This corresponds to the 'last phase' N–S compression reported by Letouzey and Tremolieres (1980) from southern Israel. Also, Mart and Horowitz (1981, table I) conducted an extensive study along the southern part of the Dead Sea rift and found a regional N–S-trending joint system along the western rift shoulder.

Joints from set O-Y3 strike in a wide azimuth range, from 012° to 035°, and they show a systematic counterclockwise joint rotation from older to younger layers of the Horsha Formation. This structural feature seems to be associated with local synclinal stresses (Bahat, 1986). Hence, the fracture characteristics shown by set O-Y3 are quite different from those of the N–S-trending joints at station X, suggesting an association with two distinct stress fields.

The joint trends and en échelon characteristics of both the sets represented by part D from station W. S. (in Wadi Sécher) and the mixed stepping from station T (some 500 m north of Wadi Sécher) are quite different from the N–S set from station X (Table 1), suggesting no palaeostress affinities between the former two and the latter. On the other hand, a fracture study at 30 stations along Wadi Sécher has revealed strong indications of joint rotation which shows certain resemblences to the fracture features of set O-Y3 (in Wadi Naim). The relationship between the structures which occur at stations O-Y3, W. S. and T are currently under investigation. Hence, two of the three styles of en échelon segmentation which can be distinguished in the Beer Sheva syncline are correlated with two distinct tectonic associations. The structural affinity of the third group to this scheme is not yet clear.

## EN ÉCHELON SEGMENTATION IN FRINGES ALONG JOINTS IN LAYERED ROCKS BY THE MECHANISM OF FRACTURE SLANTING UNDER PLANE-STRESS CONDITIONS

It was suggested that in analogy to fracture behaviour in certain engineering materials (e.g. steels), en échelon cracking occurs due to fracture slanting under plane-stress conditions (FSPS), along and close to layer boundaries, when subject to limited overburden (Bahat, 1991b).

This model was compared to others (Engelder *et al.*, 1993, p. 235) and a further elaboration of its various aspects may help to clarify certain ambiguities. The mechanism of fracture slanting provides an experimental supplementary basis for the generally accepted theory that en échelon segmentation reflects a mixed mode I + III operation (Sommer, 1969; Lawn and Wilshaw, 1975; Pollard *et al.*, 1982).

*Experimental studies*

Various fracture properties of both edge and central cracks were investigated in metallic plates as a function of their thickness (Bank-Sills and Schur, 1989). It was found that, for both geometries, as thickness increases the ratio of critical stress for the slant crack/critical stress for the flat crack increases at failure, and the crack propagation rate for the slant crack is slower than that for the flat crack. Bank-Sills and Schur (1989) observed that for both edge and central slant cracks the $K_{III}/K_I$ ratio increases as thickness decreases and that the greatest value of this ratio occurs at the plate surface, and slant cracks in thinner plates will propagate more rapidly than those in thicker ones. That is, as thickness increases the tendency for slanting decreases, and this tendency becomes greater in thinner plates.

These experimental results reaffirm previous observations (Hertzberg, 1976; Broek, 1984) that cracks in tensile fields under fatigue loading propagate out of their plane in thin plates, and they also show that this property is thickness dependent. The tendency for slanting which is associated with the increase in the $K_{III}/K_I$ ratio becomes greater with the decrease in thickness of the plate and from the centre of the plate towards its edges.

Cooke and Pollard (1996) investigated crack segmentation in polymethyl methacrylate (PMMA) under various $K_{III}/K_I$ conditions. They found that both the angle of twist of the en échelon cracks and the number of segments formed increase with an increase in the ratio $K_{III}/K_I$.

*Geological implications*

In geological terms, the $K_{III}/K_I$ ratio and the tendency for crack slanting/segmentation is at a minimum at the centre of thick layers and increases with the thinness of the layer. This explains why thin chalk layers display breakdowns to en échelon segmentation and thick chalk layers do not (Bahat, 1991b). The gradual increase in the $K_{III}/K_I$ ratio from the layer centre to its boundaries explains why crack slanting starts on the parent joint when the central plume branches, and curved, twisted barbs partly overlap each other as they propagate towards the layer boundaries (Figs 3 & 6b).

Slanting of the barbs becomes more intense towards these boundaries, where it is manifested as en échelon segmentation. The transition from the plume to en échelon segmentation has been shown by many authors (e.g. Hodgson, 1961). The gradual increase in the $K_{III}/K_I$ ratio from the layer centre to its boundaries shows that there is an inherent tendency for slanting in thin layers, and explains why thinner layers of a given rock have a greater tendency to develop plumes on joint surfaces (Roberts, 1961).

The ratio $K_{III}/K_I$ increases with the angle of twist of the en échelon cracks (Cooke and Pollard, 1996). Accordingly, the $K_{III}/K_I$ ratio should generally have been greater in association with discontinuous breakdowns, and smaller for the continuous breakdowns, along the curved joint (Fig. 6c). Furthermore, there is a greater number of segments underneath the elliptical perimeter compared to other regions on the joint surface; which also corresponds to a greater $K_{III}/K_I$ ratio underneath the elliptical perimeter during segmentation (Cooke and Pollard, 1996). Accordingly, it can be argued that slanting had been maximum along the boundary of the early elliptical perimeter and it was reduced at the distal sides when the continuous breakdown started. The centre of the elliptical perimeter is not only the locus of high $K_{III}/K_I$ ratio. The stress intensity is not uniform around the ellipse, but $K_I$ becomes greater with a decrease in the elliptical radius, intensifying the fracture process and its velocity at this location (Irwin, 1962).

Quite possibly, due to the latter reasoning, a greater $K_{III}/K_I$ ratio underneath the curved semi-ellipse of the joint from station R (Fig. 7), at the time of breakdown, should also explain why the segments close to the centre of the curved shoulder reach the layer boundary, whereas away from the centre they seem to arrest before reaching the boundary.

There is an increase in the twist angle along the dip of part D from the middle of the layer along the continuous breakdown towards the lower layer boundary (Fig. 5) which corresponds to a gradual increase in the $K_{III}/K_I$ ratio in this direction.

The new linear correlation between the length of en échelon segments and layer thickness in chalk (Fig. 10) shows that the width of a fringe of en échelon cracks caused by discontinuous breakdown increases with the layer thickness. That is, the relative size of the fringe zone which fractures under plane-stress conditions (Hertzberg, 1976; Broek, 1984; Bahat, 1991b) increases with an increase in the layer thickness; corresponding to an increase in the $K_{III}/K_I$ ratio from the centre of the layer to its boundaries (Bank-Sills and Schur, 1989).

*En échelon segmentation under constraints*

En échelon segmentation occurs under favourable conditions for mode III operation and it is limited when constraints are imposed on this mode. As was mentioned above, en échelon segmentation is very common in rocks. It is also a frequent occurrence in fractured polycrystalline metals (Hertzberg, 1976; Tschegg, 1983; Broek, 1984 and references therein) and in single crystals (Gilman, 1958). On the other hand, en échelon segmentation is rare in (non-polymeric) glass, and the widely cited experiment by Sommer (1969) is related to segmentation in glass under fluid pressure.

The explanation of these differences is that defects in the structure continuity, such as screw dislocations in single crystals (Gilman, 1958) and inclusions and grain boundaries in steel (Tschegg, 1983) and in rocks, induce segmentation. The much smaller population of such discontinuities in non-polymeric glass limits en échelon segmentation in this material. En échelon segmentation does occur in polymeric glasses such as PMMA (e.g. Cooke and Pollard, 1996). However, fracture propagation in this material must involve an early strain and a breakdown of craze fibrils (Maccagno and Knott, 1989).

The effect of 'mode III crack closure' (Tschegg, 1983) imposes an important constraint on segmentation. Namely, en échelon cracking along layer boundaries would be minimized under compression and, therefore, even if found in rocks at considerable depths an early failure at shallow depths would be suspected.

*The influence of ductile conditions on en échelon segmentation*

Segmentation is promoted by ductility. The plastic zone of mode III loading is several times larger than that for mode I for similar materials and at the same stress intensity values (Tschegg, 1983). Furthermore, singularities like inclusions are more active in ductile materials (Ritchie *et al.*, 1982). These observations coincide well with the plane-strain–plane-stress relationship in a plate showing that the plastic zone gradually increases from the plane-strain size in the interior to a plastic zone of a larger plane-stress size at the surface of the plate (Hertzberg, 1976; Broek, 1984). In analogous fashion, in geological terms, conditions along the layer boundaries are more conducive for segmentation than in the layer interior.

*Possible wider scopes of the mechanism of fracture slanting*

The FSPS model for en échelon segmentation is limited to fringes along joints in layered rocks, and it does not apply to en échelon segmentation in massive rocks like granite. Its extension to other settings of stratified structures, such as segmentation of large igneous dykes near the ground surface (Anderson, 1951) or en échelon fracturing on the ground along branches of major strike-slip faults (e.g. Allen *et al.*, 1972), however, should not be ruled out.

## SUMMARY


(1) This study focuses on various styles of dilatant out-of-plane en échelon cracks that are associated with layer restricted burial joints that cut slightly deformed cover sediments in the Beer Sheva syncline.

(2) Both twisted and gradual breakdown styles leading to en échelon segmentation were identified. Previous experimental results have shown that whereas twisting segments reflect discontinuous processes associated with abrupt changes in I + III loadings, gradual breakdowns imply continuous processes associated with gradual mixed mode loading as the joint propagates and segmentation occurs.

(3) The timing relationships of plumes, undulations and en échelon segmentation may be used as a tool to study subtle crack processes. In one instance there are reasons to suspect that although the initial breakdowns into a series of adjacent segments on a joint surface had not been synchronous, their growths terminated synchronously.

(4) Lithological properties of the rocks influence the breakdown process. Stress along welded contacts between chert and chalk induced mode III shear which led to segmentation; such that a joint in the chert (the more competent rock) transformed into en échelon cracks in the chalk.

(5) There is a positive linear fit of en échelon length to layer thickness ($R^2 = 0.928$) in fringes that were formed by discontinuous breakdowns in joints cutting chalks along Wadi Naim.

(6) The breakdown of joints into en échelon cracks in chalk layers is influenced both by local conditions and remote stresses. These two influences may be distinguished by outcrop criteria.

(7) The local conditions control properties such as: the continuous vs discontinuous breakdowns of neighbouring joints; the size ratio of en échelon cracks to layer thickness; the change in the sense of rotation of overlapping barbs on the joint surface; the opposite segmentation on curved fringes; and the synchronized propagation of undulations, plumes and en échelon cracks.

(8) The identification of remote stress directions can be aided by the analyses of: abrupt decoupling of the fringe from either unilateral or bilateral plumes; the uniformity and consistent patterns in the sense of rotation, sizes and shapes of en échelon cracks; and by addressing cases where a set of joints demonstrates a gradual azimuth rotation and the en échelon cracks at the fringes of these joints rotate in covariance with the joints. Particularly strong cases that can be linked to specific remote stresses are those which relate to fringes both below and above the parent joint and those which maintain uniform properties of all segments.

(9) There are two distinct cases of opposite senses of segment rotation in a single fringe: first, in a curved fringe; and, second, in a straight fringe. The former can best be assigned to local deviations of the minimum principal direction from perpendicularity to the parent joint, whereas the latter appears to reflect changes in remote stresses.

(10) Occasionally discoid joints display a transformation from a right-lateral shear to a left-lateral shear through a transitional zone of 'pure tension' in a curved fringe.

(11) Statement 9 should be challenged when a straight fringe shows a transformation similar to that mentioned in statement 10.

(12) The combination of en échelon segmentation and rotation styles into the analysis of regional joint distribution can considerably improve confidence in the division of joints into sets and so upgrade the analysis of palaeostress directions.

(13) Three sets of en échelon cracks emerge when grouping together the orientation and key properties of segments from various joint fringes in the Beer Sheva syncline.

(14) Two of the three sets of en échelon segmentation

which were distinguished in the Beer Sheva syncline are correlated with two distinct tectonic associations. The structural affinity of the third group to this scheme is not yet clear.

(15) The mechanism of fracture slanting provides an experimental supplementary basis for the generally accepted theory that en échelon segmentation reflects a mixed mode I + III operation.

(16) It is inferred from the slanting fracture model that the $K_{III}/K_I$ ratio increases from the layer centre towards the layer boundaries, and this is the driving motivation for both the trend of gradual increase of barb overlapping from the centre towards the layer boundaries and the en échelon segmentation along the layer boundaries.

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